

**COMMON TRENDS AND COMMON CYCLES
AMONG INTEREST RATES
OF THE G7-COUNTRIES**

Nannette Lindenberg and Frank Westermann

Working Paper 77
January 2009

INSTITUTE OF EMPIRICAL ECONOMIC RESEARCH
University of Osnabrück
Rolandstrasse 8
49069 Osnabrück
Germany

Common Trends and Common Cycles among Interest Rates of the G7-Countries*

Nannette Lindenberg*

Frank Westermann

January 19, 2009

Abstract

In this paper we re-investigate the comovements of interest rates in the G7-countries. We propose a structured *modus operandi* to analyze the time series characteristics of interest rates and to test for common features. We conduct cointegration, serial correlation common feature and codependence tests with nominal and real interest rates using quarterly data from 1975 to 2007. Overall we only find little evidence of comovements. Common trends are occasionally observed, but the majority of interest rates are not cointegrated. Although some evidence for codependence of higher order is found among European countries, common cycles appear to exist only in rare cases and cannot be generalized for all interest rates.

Keywords: Interest Rates, Comovement, Cointegration, Serial Correlation Common Feature, Codependence

JEL: C22, E43, G12

*We would like to thank Yin-Wong Cheung as well as seminar participants at the University of Osnabrück for helpful discussions and comments.

*Corresponding Author: Institute for Empirical Economic Research, University of Osnabrück, Rolandstraße 8, 49069 Osnabrück, Germany, Email: nannette.lindenberg@uos.de

1 Introduction

A recent empirical literature on interest rates analyzes whether they are integrated in the short-run and in the long-run, using different concepts and types of interest rates.¹ Methods to test for comovement include the tests for *cointegration*, *serial correlation common feature* and *codependence*.² Results from these studies, however, are sometimes difficult to reconcile, as the various tests are appropriate for different types of stochastic properties of the data series. In particular, they depend on the stationarity or non-stationarity, as well as on the lag structure of the time series under investigation.

We contribute to the existing literature by implementing a structured framework to analyze possible comovements in interest rates. First, we analyze the time series characteristics of interest rates. We start by identifying the AR(p)-structure and testing for stationarity, taking into account the small sample properties of the data set.³ Then, in the set of non-stationary interest rates, we test for common trends, using the Johansen (1988, 1991) cointegration procedure.⁴ Finally, among the set of stationary interest rates, we test for common cycles, using the serial correlation common feature and the codependence tests, that were initially suggested by Engle and Kozicki (1993) and Vahid and Engle (1997). For the latter test we employ two different methods: the Two-Stage-Least-Square (TSLS) estimation, and the optimal GMM estimation proposed by Cubadda (1999).

We apply our analysis to money market rates, government bond yields and Euro-Market rates, both nominal and real, using a long sample of quarterly data from 1975 to 2007. The idea behind choosing these series is to include interest rates of different issuers (public, private), different maturities, and rates with and without country risk. Our main conclusion is that, independent of the interest rate measure that is chosen, there is only very limited evidence on either long-run or short-run comovements in the G7-countries.

In the set of non-stationary interest rates we find some evidence of cointegration, but the majority of interest rates are not cointegrated. Among stationary interest rates, we find even less evidence of comovements. As a first pass, we observe that the lag structure of the AR-representations varies substantially across countries, which indicate that it is unlikely to find a common cyclical pattern among interest rates. More formally, we reject the strict form of a common serial correlation feature for all interest rates, and find evidence of codependence - a common cyclical pattern after an initial time interval - only in rare cases. Taking various efforts to find more positive evidence of short-run comovements, such as changing the sample period, lag structures, and estimation procedures, does not change this main conclusion. Some evidence of

¹Such as Poghosyan and de Haan (2007); Romero-Ávila (2007); Zhou (2003); Bremnes, Gjerde and Soettem (2001); Kugler and Neusser (1993).

²Another possible method is to directly test for uncovered or covered interest rate parity. See for example Chinn and Meredith (2004) who test UIP for the G7-countries using short and long horizon data and find evidence for UIP in the latter case.

³We find that interest rates are in most cases I(1) (as in Romero-Ávila (2007); Rapach and Weber (2004); Bremnes et al. (2001); King, Plosser, Stock et al. (1991); Rose (1988)). Although some series are also I(0) (as in Choi and Chul Ahn (1999); Wu and Zhang (1996); Kugler and Neusser (1993)). This finding is consistent with studies that conclude that there is no unambiguous evidence for either stationarity or non-stationarity for all interest rates (as in Cheung, Tam and Yiu (2008); Koustas and Lamarche (2007); Karanasos, Sekioua and Zeng (2006)).

⁴Again, we control for finite sample properties, using the scaling factors of Cheung and Lai (1993).

higher order codependence is found, however, among European countries, France, Italy and the UK.

With regard to the long-run comovements among interest rates that were previously reported in the literature, we confirm the finding that cointegration is found only in the minority of the cases (Romero-Ávila (2007); Zhou (2003); Bremnes et al. (2001)) and in special circumstances (Poghosyan and de Haan (2007)). Concerning comovements in the short-run, our results suggest that more positive evidence reported in Kugler and Neusser (1993) cannot be generalized for all interest rates, time periods, and reasonable alternative estimation procedures.

The paper is organized as follows: The next section presents the data description and preliminary analysis, namely unit root tests and the definition of the autoregressive process of the interest rates. Section three discusses comovements in interest rates. After a brief presentation of the applied methodology, we then present results of the cointegration, serial correlation common feature and codependence tests for nominal and real interest rates. In order to augment the possibility of finding at least common if not synchronized cycles we use two different methodologies for the latter common feature. The last section contains some concluding remarks.

2 Data and Descriptive Statistics

2.1 Description of the data

Our analysis is conducted with nominal and ex post real interest rates for the G7-countries: Canada, France, Germany, Italy, Japan, the United Kingdom and the United States. We use money market rates and 10-year government bond yields. In addition, we run the same investigation with Euro-Market rates of the seven countries. Our sample period covers quarterly data of the post Bretton-Woods-era, i.e. from 1975:1 to 2007:1.

For the analysis of real interest rates, ex post rates are constructed with the logged first differences of the respective consumer price index.

Data on money market rates, government bond yields and consumer prices are extracted from the International Financial Statistics Database of the International Monetary Fund. In order to complement unavailable data on French interest rates, missing data are taken from the statistics of the Banque de France. The Euro-Market rates are provided by Global Insight (see table A.1 in the appendix for a detailed data description).

Graphs of the nominal interest rates are displayed in figure 1 in the appendix.

2.2 Preliminary Analysis

We start our empirical analysis by defining the lag structure of each time series and by performing unit root tests.

Process We first estimate different AR representations of the process of each variable using the following regression equation:

$$x_t = \mu + \sum_{i=1}^p \beta_i x_{t-i} + \epsilon_t,$$

with x_t = the interest rate at time t , p = lag parameter and ϵ_t = an error term. We then select a parsimonious lag structure⁵ by identifying the minimum lag length that is needed to remove all autocorrelation from the residuals, i.e. all Q-statistics are insignificant.

In addition, we perform the same analysis with the first differences of the interest rates Δx_t . The lag structures of the nominal and real interest rates and their first differences are shown in table 1.

The lag structures of the nominal interest rates typically vary from AR(1) to AR(6).⁶ Similarly, the real interest rates have lag structures from AR(1) to AR(8). The majority of the nominal interest rates are found to have an AR(2) representation, whereas the lag structure of the real interest rates is more uniformly distributed between the different possibilities in the range mentioned above.

The diversity of lag structures across countries and interest rates is interesting in the context of the main aim of the paper, that is to identify common trends and common cycles among interest rates. While section three will formally investigate whether the autoregressive components of interest rate time series ("cycles") are common across countries, the diversity of lag structures in interest rates already tells us, that an important precondition for finding common cycles (or common serial correlation patterns) is not met in many countries. Not being able to establish a common lag structure in many cases, makes it very unlikely to find strong evidence in favor of a common serial correlation feature in the following exercises.

Stationarity As a second preliminary exercise, we test for possible unit roots, using the Augmented-Dickey-Fuller (ADF) test:⁷

$$\Delta x_t = \mu + \beta t + \gamma x_{t-1} + \sum_{j=1}^p \phi_j \Delta x_{t-j} + \epsilon_t,$$

where Δ is the first difference operator and t a time trend.⁸

⁵In order to generate the best condition for finding serial correlation common features or codependence we choose the most parsimonious model. However, using the AIC or SIC criterion to choose the lag length does not change the results qualitatively.

⁶There is an exceptionally high lag length of 12 for the money market rate in Germany.

⁷In a first round we allow for an intercept and a time trend, and if the time trend is insignificant, we conduct the ADF-test in a second round only with an intercept.

⁸The lag parameter p is determined by the Akaike information criterion (AIC). Using the Schwarz criterion (SIC) gives in some cases different results. However, the qualitative interpretation of the analysis is robust to the selected lag length. The subsequent tests for comovements will be applied to all interest rates that are I(0) (respectively I(1)) indicated by either one of the two criteria. A comparison of the results of the ADF-test with either AIC or SIC is available in the unpublished appendix.

Table 1: AR(p) representations of the process of nominal and real interest rates

1975:1 - 2007:1 (1979:1 - 2007:1 for Euro-Market Rates)				
Country	Nominal		Real	
	AR(...)		AR(...)	
	level	1st diff	level	1st diff
Money Market Rates				
Canada	6	6	8	8
France	2	1	3	2
Germany	12	2	1	1
Italy	2	1	3	3
Japan	4	2	1	4
UK	1	4	5	8
USA	6	5	7	2
Government Bond Yields				
Canada	4	3	2	4
France	2	1	5	4
Germany	4	3	1	1
Italy	2	1	5	4
Japan	2	1	5	3
UK	1	3	8	5
USA	4	3	2	1
Euro-Market Rates				
Canada	1	3	1	3
France	2	2	2	4
Germany	2	1	1	1
Italy	5	4	5	4
Japan	4	3	2	1
UK	6	4	5	2
USA	4	2	3	2

Note: AR representations of the process of the nominal and real interest rates (money market rates, government bond yields and Euro-Market rates) for the sample 1975:1 to 2007:1 (respectively from 1979:1 onwards for the Euro-Market rates) are reported. The specification with the smallest number of AR terms is selected, under the constraint that the residual is free of autocorrelation (i.e. the Q-statistics are insignificant).

The t-statistic of the ADF-test is compared with the finite sample critical values from Cheung and Lai (1995).⁹

We apply the ADF-test to the levels and to their first differences. Results of the ADF-test are shown in table 2. For all nominal interest rates the null hypothesis of a unit root cannot be rejected in levels, but in first differences it can. Thus, the nominal interest rates are I(1) series.¹⁰

For the real interest rates the picture is quite different. The money market rates of Canada and the USA as well as the government bond yields of Germany, Italy and the USA are stationary. This amounts to about 25% of all time series under investigation. For the remaining interest rates we cannot reject the null of non

⁹The main conclusions remain the same whether we use the critical values of MacKinnon (1996) or the finite sample critical values of Cheung and Lai (1995).

¹⁰About a third of the series are trendstationary. For the purpose of our paper, we treat these interest rates as non-stationary and do not further distinguish between deterministic and stochastic trends.

Table 2: Results of ADF-test for nominal and real interest rates

1975:1 - 2007:1 (respectively 1979:1 - 2007:1 for Euro-Market Rates)				
Country	Nominal Interest Rates		Real Interest Rates	
	Level	1st diff.	Level	1st diff.
Money Market Rates				
Canada	-3.463	-5.186 **	-3.330 **	-6.351 **
France	-2.995	-6.611 **	-2.185	-6.718 **
Germany	-2.647	-4.799 **	-2.54	-5.742 **
Italy	-1.945	-7.871 **	-2.089	-7.832 **
Japan	-4.002	-5.188 **	-4.729	-7.759 **
UK	-1.985	-4.695 **	-2.487	-4.118 **
USA	-3.613	-4.765 **	-3.063 **	-5.925 **
Government Bond Yields				
Canada	-2.366	-4.736 **	-2.602	-8.382 **
France	-2.25	-5.863 **	-1.533	-5.808 **
Germany	-3.323	-7.848 **	-2.942 **	-5.257 **
Italy	-2.633	-7.248 **	-3.808 **	-9.845 **
Japan	-3.298	-5.058 **	-6.462	-7.098 **
UK	-3.905	-9.736 **	-2.176	-5.060 **
USA	-2.959	-5.742 **	-3.221 **	-8.770 **
Euro-Market Rates				
Canada	-3.667	-10.66 **	-4.347	-5.205 **
France	-3.806	-4.159 **	-3.966	-4.363 **
Germany	-3.029	-3.500 **	-3.754	-9.691 **
Italy	-3.523	-3.432 **	-5.405	-4.942 **
Japan	-2.626	-8.531 **	-1.657	-8.797 **
UK	-3.832	-3.599 **	-2.6	-6.525 **
USA	-3.394	-3.765 **	-2.35	-2.734

Note: The ADF-test statistics, calculated for the levels and first differences of nominal and real interest rates (money market rates, government bond yields and Euro-Market rates) for the sample 1975:1 to 2007:1 (respectively from 1979:1 onwards for the Euro-Market rates), are reported. The lag length was selected by the AIC criterion. Critical values of Cheung and Lai (1995) were applied. ** indicate rejection of the existence of both, stochastic and deterministic trends with a significance of 5%.

stationarity.

Again, the mixed pattern found on stationarity suggests that strong evidence on either common trends or common cycles is unlikely to be found in this data set.

3 Comovements in Interest Rates

In recent years a number of new tests to detect comovements in time series have been developed (see Urga (2007) for a discussion). The concept of common feature has been first suggested by Engle and Kozicki (1993) and encompasses common stochastic trends (the $I(1)$ -feature), common serial correlation (common $AR(p)$ -feature), as well as other times series characteristics, including seasonality and ARCH and GARCH effects.

A necessary element in the analysis is that the "feature" ($I(1)$, $AR(p)$, and so on) needs to exist in *both* time series under investigation. The large diversity of $AR(p)$ processes and $I(1)/I(0)$ processes in interest

rate data documented above yields only a limited number of country pairs where the common features of interest - common trends and common cycles - can be tested.

We divide the following analysis into two parts: with the interest rates that are found to have a unit root we conduct cointegration tests and with the stationary interest rates we pursue serial correlation common feature and, if applicable, codependence tests.

3.1 Common Stochastic Trends

To test for cointegration we adopt the Johansen (1988, 1991) maximum likelihood approach, allowing for an intercept in the cointegrating equations. We assume that the time series that we analyze for cointegration follow a vector autoregressive process of order p :

$$X_t = \mu + \sum_{i=1}^{p-1} \Gamma_i X_{t-i} + \epsilon_t$$

where X_t is a $n \times 1$ vector of different interest rates (with n = number of interest rates included in the VAR), μ is an intercept vector and ϵ_t is a vector of error terms.

In order to generate the test statistic of the Johansen test, namely the trace or the maximum eigenvalue statistic¹¹, we require the canonical correlations between the least squares residuals of the two subsequent regressions:

$$\begin{aligned} \Delta X_t &= \mu_1 + \sum_{i=1}^{p-1} \Gamma_i \Delta X_{t-i} + \epsilon_{1t} \\ \text{and } X_{t-p} &= \mu_2 + \sum_{i=1}^{p-1} \Gamma_i \Delta X_{t-i} + \epsilon_{2t}, \end{aligned}$$

i.e. between the matrices ϵ_{1t} and ϵ_{2t} . For the lag parameter p we choose the largest lag structure of the process of the first differenced terms of the relevant interest rates (see table 1).

The maximum eigenvalue statistic tests the null hypothesis of r cointegrating vectors against the alternative hypothesis that there are $r + 1$ cointegrating vectors:

$$\text{Maximum Eigenvalue Statistic} = -T \ln(1 - \lambda_{r+1}).$$

The number of cointegration relations r is restricted to $0 \leq r \leq n$ with n being the number of variables.

¹¹In the subsequent discussion of the results of the cointegration tests only the maximum eigenvalue statistic will be used. The trace statistic tests the null hypothesis that there are r or fewer cointegrating vectors:

$$\text{Trace Statistic} = -T \sum_{k=r+1}^n \ln(1 - \lambda_k),$$

where λ_k are the squared canonical correlations calculated in the former step of the analysis. The trace statistic gives slightly different interpretations in some cases. However, a pattern that one of the two statistics is more favorable to cointegration cannot be detected. Comparison tables with both test statistics (trace and maximum eigenvalue) are available upon request.

$n - r$ assigns the number of independent stochastic trends pushing the long-run dynamics.

The test statistic is compared with the critical values of Osterwald-Lenum (1992) for the 5% and 1% levels that are corrected with the scaling factor of Cheung and Lai (1993) to control for a possible finite-sample bias.

We perform tests with all available pairs of interest rates. This yields up to $\binom{7}{2} = 21$ possible cointegration relations (as we have seven different countries) among each type of interest rates.

Nominal Interest Rates We start our analysis with the nominal money market rates, nominal government bond yields and nominal Euro-Market rates.

Among the nominal interest rates we do not find any stationary time series, so that we can conduct the cointegration analysis with all of the nominal interest rates. The results of these tests are shown in table 3.

The evidence shows that less than the half of the pairs of nominal interest rates are cointegrated: The null hypothesis of no cointegration is rejected at the 1% level for the money market rate of Italy with the French, and the US money market rates, as well as for the British-French, the British-German and the British-Japanese pairs. Furthermore, the money market rates of France and Japan; France and the USA; and Japan and Italy reject the null hypothesis of no cointegration at the 5% level of significance.

Much less cointegration relation is revealed between the government bond yields of the different countries. Only for the Canadian-German, the Franco-American and the Italian-German pairs of bonds the null of no cointegration can be rejected at the 5% level of significance. Overall, bivariate cointegration is found for 38.1% (14.3%) of the nominal money market rates (government bond yields).

Running the Johansen test with the nominal Euro-Market rates yields only one cointegrated pair of interest rates: The Euro-Market rates of France and the USA reject the null of no cointegration at the 1% level of significance.

Real Interest Rates One possible distortion to comovements among interest rates are differing inflation rates. Therefore, we conduct the same analysis also for real interest rates.¹² However, we find that the results are overall very similar.

We reject the null of no cointegration at the 1% level of significance and find cointegration between the real money market rates of Germany and Italy; Germany and Japan; Japan and the UK; and Japan and the USA.¹³ Furthermore, the Franco-German, and the Franco-Italian pairs of real money market rates reject the null hypothesis of no cointegration at the 5% level.

Among the real government bond yields we detect five more pairs that are cointegrated (rejection at the 1% level of significance): the Canadian-Japanese, the Canadian-UK, the Franco-German, the Franco-

¹²For the cointegration test of the real interest rates we exclude those interest rates that are found to be unambiguously stationary, namely the money market rate of Canada, and the government bond yields of Italy and the USA. Note that the decision for or against non-stationarity sometimes varies depending on the selected lag length.

¹³The results of the Johansen test for real interest rates are shown in the unpublished web appendix.

Table 3: Results of Johansen Test for nominal interest rates

1975:1 - 2007:1 (respectively 1979:1 - 2007:1 for Euro-Market Rates)							
Johansen Test (Maximum Eigenvalue Statistic) of Money Market Rates							
	Canada	France	Germany	Italy	Japan	UK	USA
Canada	--						
	--						
France	r=0 19.832	--					
	r=1 1.4049	--					
Germany	r=0 9.8543	8.7667	--				
	r=1 4.1231	2.2426	--				
Italy	r=0 25.463	22.454 ***	10.005	--			
	r=1 1.4137	3.3558	2.0208	--			
Japan	r=0 19.969	18.694 **	11.203	17.683 **	--		
	r=1 3.859	5.3931	8.7638	4.3105	--		
UK	r=0 10.109	21.812 ***	25.216 ***	14.371	22.308 ***	--	
	r=1 4.6703	1.9755	4.9939	1.1873	3.0814	--	
USA	r=0 10.896	19.653 **	12.277	25.103 ***	12.571	12.929	--
	r=1 2.1592	2.042	5.742	1.2217	5.2862	6.6093	--
Johansen Test (Maximum Eigenvalue Statistic) of Government Bond Yields							
	Canada	France	Germany	Italy	Japan	UK	USA
Canada	--						
	--						
France	r=0 15.996	--					
	r=1 1.6877	--					
Germany	r=0 18.992 **	13.911	--				
	r=1 0.9724	1.5174	--				
Italy	r=0 11.394	9.5678	18.864 **	--			
	r=1 1.3328	1.7641	1.4064	--			
Japan	r=0 11.139	7.8719	12.059	13.865	--		
	r=1 2.1828	3.6399	2.7951	4.6457	--		
UK	r=0 14.551	9.6901	11.555	13.04	13.347	--	
	r=1 3.5876	6.0517	3.7901	5.1034	5.7657	--	
USA	r=0 6.8735	20.177 **	15.563	12.135	11.367	13.643	--
	r=1 1.0324	2.0187	2.3973	1.6545	2.8818	4.4668	--
Johansen Test (Maximum Eigenvalue Statistic) of Euro-Market Rates							
	Canada	France	Germany	Italy	Japan	UK	USA
Canada	--						
	--						
France	r=0 9.9951	--					
	r=1 4.1521	--					
Germany	r=0 6.7625	7.8962	--				
	r=1 4.1075	3.6335	--				
Italy	r=0 17.012	28.34	8.278	--			
	r=1 3.5435	1.7724	2.5586	--			
Japan	r=0 21.57	41.06	22.288	26.266	--		
	r=1 3.0846	2.8414	3.0897	19.647	--		
UK	r=0 14.089	20.115	16.252	14.032	31.166	--	
	r=1 8.551	3.3103	4.923	4.4982	8.2539	--	
USA	r=0 6.9337	25.112 ***	7.3094	15.145	9.5507	3.5474	--
	r=1 5.7634	3.4048	4.106	6.8402	3.8312	2.9644	--

Note: Results of testing for bivariate cointegration among the nominal interest rates (money market rates, government bond yields and Euro-Market rates) for the sample 1975:1 to 2007:1 (respectively from 1979:1 onwards for the Euro-Market rates) are shown. The table contains the Maximum Eigenvalue statistics for $r=0$ and $r=1$ for each pair of variables. The critical values of Osterwald-Lenum (1992) were scaled with the scaling factor of Cheung and Lai (1993) to adjust for finite samples. ** and *** indicate the rejection of the null hypothesis with a significance of 5% and 1%.

Japanese and the German-Japanese couple. The Canadian-German pair of real government bond yields still rejects the null hypothesis of no cointegration at the 5% level.

Summing up, with 6 out of 21 possible pairs being cointegrated (thus 28.6%) the change from nominal to real interest rates does not enhance the evidence for cointegration among money market rates and government

bond yields in a significant way. Among the real Euro-Market rates there is no evidence for cointegration.

3.2 Common Cycles

Serial Correlation Common Feature As a generalization of the cointegration test, Engle and Kozicki (1993) have developed the test for common features. If a feature (such as the I(1)-property, AR(p)-structure, seasonality, etc.) is common to two times series, there should exist a linear combination that does not have this feature.

In this section, we focus on the common serial correlation feature (SCCF). While the previous section focused on lon-run comovement, the analysis of a common autoregressive structure of time series is often interpreted as cyclical comovement (Cheung and Westermann (2002)). It is important to point out, that not *both* tests can be meaningfully applied to the same time series, as one requires stationarity, and the other non-stationarity. In this section we therefore only continue with interest rates that were identified as I(0)-series in table 2.¹⁴

Furthermore, a "common feature" among time series can only exist, when the feature (i.e. the AR(p)-structure) is identical in both time series. As reported in table 1, this is rarely the case, however, and, as pointed out in section 2.2, this substantially limits the degree of common cycles that we anticipate to find in the data.

For those pairs of interest rates that are stationary and follow the same stochastic process we estimate the equation

$$x_t = c + \beta y_t + \epsilon_t \quad (1)$$

with two-stage-least-squares, including as instruments all lagged variables of x and y , i.e. x_{t-k} and y_{t-k} for $k = 1, \dots, p$. $(1, \beta)$ is the normalized common feature vector.

In a second step, we test whether the estimated residual $\hat{\epsilon}_t$ of the former estimation is still driven by the same stochastic process as x_t and y_t . Therefore, we estimate the following equation by OLS:

$$\hat{\epsilon}_t = c + \sum_{k=1}^p \delta_k x_{t-k} + \sum_{k=1}^p \gamma_k y_{t-k} + u_t. \quad (2)$$

Next, we test the null hypothesis that all lagged variables of x_t and y_t do not explain jointly the endogenous variable $\hat{\epsilon}_t$, i.e. $\delta_k = \gamma_k = 0$ for $k = 1, \dots, p$. If the lagged variables do not explain the movement of the estimated residual, the common AR(p)-pattern of the interest rates x_t and y_t is removed. The null hypothesis is tested with the F-statistic:

¹⁴It is also possible to conduct the serial correlation common feature test in the 1st differences of the non-stationary interest rates. However, as it is difficult to attribute an economic interpretation to the change in interest rates, we focus on the levels of interest rates in this paper only.

$$F_{k-1, T-k} = \frac{R^2}{1 - R^2} \frac{T - k}{k - 1},$$

where T denotes the number of observations and k refers to the number of restrictions, i.e. the number of exogenous variables including the constant. R^2 is the R-squared of regression 2. The null hypothesis of a common feature will be rejected, if the value of the calculated F-statistic is larger than the tabulated critical value of the F-distribution.¹⁵

For the nominal interest rates we cannot perform the analysis of testing for serial correlation common features as we do not have a single pair of stationary interest rates.¹⁶ Thus, none of the potentially 63 pairs of nominal interest rates shares a serial correlation common feature.

Among the real interest rates there exist several pairs of interest rates that meet the requirements of being stationary and following the same AR process.¹⁷ The results of the serial correlation common feature test are displayed in table 4 (panel A).

In our sample, 10 of the 21 real interest rates fulfill the condition of being stationary. Among these 10 time series we identify 2 pairs of interest rates of the same type that follow the same autoregressive process: the real government bond yields of Canada and the USA (AR(2)); and of France and Italy (AR(5)). For both pairs of real interest rates we reject the null hypothesis of a common feature at the 1% significance level. Thus, none of the 63 pairs of real interest rates share a common serial correlation feature.

As a robustness test, we also conduct the analysis in a shorter sample, ending in 1998:4. The aim is to control for possible influences of the change in the institutional environment that resulted from the introduction of the ECB.

In comparison to the initial sample, we find three more pairs to test for serial correlation common feature: real government bond yields of France, Italy and the UK (all AR(5)-processes) form the first three couples and the real Euro-Market rates from Canada and France (AR(1)), and from Italy and the UK (AR(2)) are the remaining two couples (see panel B of table 4). In four of the five cases the F-statistics are significant, indicating that a common serial correlation does not exist. In the case of the real Euro-Market rates of Italy and the UK, the F-statistic is insignificant. However, the cofeature vector is insignificant. Thus, a serial correlation common feature does not exist in the shortened sample either.

Codependence: TSLS Estimation In table 4 we also report the results on codependence among interest rates of higher order. This weaker form of cyclical, but non-synchronized comovement was first described by Gourieroux and Peaucelle (1989) and Vahid and Engle (1997): Some time series may have a different initial

¹⁵Thus, in contrast to the cointegration test, where a rejection of H_0 stands for the existence of a common trend, in the case of the serial correlation common feature test we must not reject the null hypothesis in order to detect a common feature.

¹⁶See table 2.

¹⁷See table 1, table 2 and comparing the table of ADF-tests with AIC and SIC in the web appendix.

Table 4: Results of serial correlation common feature and codependence tests for real interest rates

PANEL A							
1975:1 - 2007:1							
Country	AR(...)	Coefficient	CF = 0	1	Codependence of order		
				2	3	4	
Real Government Bond Yields							
Canada	2	0.7616	129.3 ***	5.251 ***			
USA	2						
France	5	0.6351	30.09 ***	3.466 ***	2.312 **	3.136 **	4.974 ***
Italy	5						
PANEL B							
1975:1 - 1998:4 (respectively 1979:1 - 1998:4 for Euro-Market Rates)							
Country	AR(...)	Coefficient	CF = 0	1	Codependence of order		
				2	3	4	
Real Euro-Market Rates							
Canada	1	0.5611	8.409 ***				
France	1						
Italy	2	0.3097	0.921	1.713			
UK	2						
Real Government Bond Yields							
France	5	0.6192	22.53 ***	2.787 ***	1.619	2.204	3.167 **
Italy	5						
France	5	0.5511	35.65 ***	5.217 ***	2.781 **	2.134	4.261 **
UK	5						
Italy	5	0.8429	24.83 ***	7.443 ***	3.372 ***	3.099 **	6.132 ***
UK	5						

Note: Results of the common feature and codependence test of real government bond yields for the sample 1975:1 - 2007:1 (panel A) are reported. In the lower part of the table we report the results of the common feature and codependence test of real government bond yields for the sample 1975:1 - 1998:4 (panel B) and of real Euro-Market rates for the sample 1979:1 - 1998:4 (panel B). Only few pairs of real interest rates are stationary and have the same autoregressive representation (indicated in the second column). The third column contains the coefficient of the common feature vector. The following columns report the F-statistics for the common feature test (= codependence of order 0) and the codependence tests. *, ** and *** indicate the rejection of the null hypothesis with a significance of 10%, 5% and 1%.

response to a shock, but a common response after some lags.¹⁸

We test for codependence estimating the same equations as for the SCCF: 1. TSLS (equation 1) and 2. OLS of the residual (equation 2).

Then, we compute a Wald-Test, testing whether all but the first lagged terms of both interest rates do not explain jointly the estimated residual $\hat{\epsilon}_t$. The null hypotheses are: H_0 for codependence of order 1: $\delta_k = \gamma_k = 0$ for $k = 2, \dots, p$; H_0 for codependence of order 2: $\delta_k = \gamma_k = 0$ for $k = 3, \dots, p$ and so forth. Again, H_0 will be rejected, if the value of the calculated F-statistic is larger than the tabulated critical value.

We test for codependence of order one for the real government bond yields of Canada and the USA and for codependence up to order four for the real government bond yields of France and Italy (see table 4 (panel A) for the results). All test statistics are highly significant, thus a codependence relationship does not exist

¹⁸Thus, codependence of order 0 is actually a serial correlation common feature.

among the country pairs.

As before, we pursue the codependence tests for the shortened sample (see table 4 (panel B)). In this specification, very weak evidence is found in favor of codependence. The result lends support to the view that the real government bond yields of France and Italy are codependant of order two and order three and that the real government bond yields of France and the UK show third order codependence. However, codependence of order four is not found in both cases, which suggests that there is little correlation left after the third lag in either time series.

To sum up, reliable evidence for a comovement in the transitory components of the interest rates cannot be found. As the evidence for serial correlation common feature is really poor among all types of interest rates, we finally ignore for a moment the condition that the time series have to follow the same process of autocorrelation and conduct the test for serial correlation common feature with all interest rates of the same type that are $I(0)$. The results are displayed in table A.2 in the appendix. This relaxation leaves us with three pairs of real money market rates and 15 pairs of real government bond yields to test for comovements in the short-run. For each test the respectively longer lag length is selected. All test statistics are highly significant (at the 1% level), meaning that no serial common feature exists among them. This result indicates that the poor evidence for serial correlation common feature is not due to a wrong lag length selection.

Codependence: GMM Estimation We finally consider a GMM estimation of the codependence relationship. Vahid and Engle (1997) and Cubadda (1999) both report that due to its relative efficiency, an optimal general method of moments (GMM) estimation is more appropriate for a codependence test than a TSLS estimation.¹⁹ Thus, in a last attempt to detect evidence for cyclical comovement, we conduct the optimal GMM test proposed by Cubadda (1999). Results of the GMM estimation of codependence relations are shown in table 5.

As in the previous section, we first test the two pairs of real government bond yields of Canada and the USA; and France and Italy, where common lag structures are given in the full sample (table 5 (panel A)). In the former case the χ^2 -statistic is highly significant. In the latter case we find only weak evidence for codependence of order four, as the null hypothesis of codependence can only be rejected at the 10%-level.

In the shorter sample the evidence for some codependence strengthens a bit: For the real government bond yields of France and Italy we find the same weak evidence for codependence of order four; and for the real government bond yields of France and the UK; and Italy and the UK we find codependence of order three (the null hypothesis of codependence cannot be rejected at conventional levels) (see table 5 (panel B)).

Concerning the real Euro-Market rates we can still not find evidence for codependence. In the case of the

¹⁹Schleicher (2007) proposes another alternative to the TSLS procedure claiming that likelihood ratio (LR) tests based on full information maximum likelihood (FIML) estimates have even higher power than the optimal GMM estimates when testing for codependence of order one. However, this applies above all if error correction terms are included, which is not given in our investigation.

Table 5: Results of optimal GMM estimation of codependence relations for real interest rates

PANEL A		1975:1 - 2007:1				
		Codependence of order				
Country	AR(...)	0	1	2	3	4
Real Government Bond Yields						
Canada	2	Vector	- 0.7101 ***	- 0.7024 ***		
USA	2	χ^2 -test	118.3175 ***	59.8897 ***		
France	5	Vector	- 0.6587 ***	- 0.6666 ***	- 0.6888 ***	- 0.7386 ***
Italy	5	χ^2 -test	115.5616 ***	54.7611 ***	28.3730 ***	13.6564 ***
PANEL B						
		1975:1 - 1998:4 (respectively 1979:1 - 1998:4 for Euro-Market Rates)				
		Codependence of order				
Country	AR(...)	0	1	2	3	4
Real Euro-Market Rates						
Canada	1	Vector	- 0.1888 **			
France	1	χ^2 -test	66.0714 ***			
Italy	2	Vector	- 0.2746 **	- 0.2875		
UK	2	χ^2 -test	41.3502 ***	21.0472 ***		
Real Government Bond Yields						
France	5	Vector	- 0.5598 ***	- 0.5731 ***	- 0.5971 ***	- 0.6594 ***
Italy	5	χ^2 -test	85.1935 ***	40.2984 ***	21.1621 ***	11.4594 ***
France	5	Vector	- 0.4206 ***	- 0.4641 ***	- 0.5692 ***	- 0.7779 ***
UK	5	χ^2 -test	86.3631 ***	41.3578 ***	19.0679 ***	5.6549
Italy	5	Vector	- 0.7591 ***	- 0.8348 ***	- 0.9261 ***	- 0.9634 ***
UK	5	χ^2 -test	79.9158 ***	30.3733 ***	8.2637 **	0.4974

Note: Results of the optimal GMM estimation of codependence relations of real government bond yields for the sample 1975:1 - 2007:1 (panel A) and 1975:1 - 1998:4 (panel B) as well as of real Euro-Market rates for the sample 1979:1 - 1998:4 (panel B) are reported. *, ** and *** indicate statistical significance at the 10%, 5% and 1% level.

Italian-UK pair, the codependence vector of order one is insignificant, but the coefficient in the co-feature relationship is insignificant.

Overall, convincing evidence for cyclical comovement cannot be found. Some codependence of higher order can be detected in a few special cases. The strongest evidence of codependence exist among the EU countries, Italy, France and the UK.

4 Conclusions

The contribution of this paper is to analyze comovements in interest rates in a structured framework. For this purpose we performed the analysis with different interest rates in a post Bretton-Woods sample: we started with nominal interest rates (money market rates, government bond yields and Euro-Market rates) and continued with real interest rates.

Already the preliminary analysis of defining the lag structure and the stationarity of the interest rates

gives rise to the suspicion that little evidence of comovement can be detected in our data set. We generally find only weak evidence for cointegration. Neither the abstraction of the country risk by looking at Euro-Market rates, nor the distinction between nominal and real rates is vital for finding comovements in the long run. The same conclusion also applies to cyclical comovement: using two different methodologies, the TSLS estimation proposed by Engle and Kozicki (1993) and a GMM estimation suggested by Cubadda (1999), we were not able to establish convincing evidence of either common synchronized cycles (SCCF) or common non-synchronized cycles (codependence). Very limited evidence of higher order codependence only exists among a few European countries.

The lesson to be drawn from our analysis is thus that we cannot assume common stochastic characteristics of interest rates, and we also cannot generalize the limited evidence of comovements, that is found in this paper, for all interest rates.

Further research could analyze whether the currency risk could explain the weak evidence for comovements among interest rates.²⁰ We tried to approach this aspect by performing the analysis for the three Eurozone-Countries: France, Germany and Italy, for the time after the introduction of the euro as common currency.²¹ Although, we did not find common cycles in this attempt either, this finding might be due to the limited number of observations.

Another promising field for future research could be to extend the test in the literature on the causality among interest rates. A deeper knowledge of the trend and cycle comovements may be useful for this purpose. Furthermore, an analysis of different tax regimes might help to explain the weak evidence of comovement.²²

Finally, it would be interesting to contrast the finding of the G7-countries to either emerging market rates, or the interest rates of large economies, with small neighboring countries, that pursue similar central bank policies.

References

- Bremnes, H., Gjerde, O. and Soettem, F. (2001), "Linkages among Interest Rates in the United States, Germany and Norway". *The Scandinavian Journal of Economics*, **103**(1), 127–145.
- Cheung, Y.-W. and Lai, K. S. (1993), "Finite-Sample Sizes of Johansen's Likelihood Ratio Tests for Cointegration". *Oxford Bulletin of Economics and Statistics*, **55**(3), 313–328.
- Cheung, Y.-W. and Lai, K. S. (1995), "Lag Order and Critical Values of the Augmented Dickey-Fuller Test". *Journal of Business & Economic Statistics*, **13**(3), 277–280.

²⁰The interaction between interest rates and the exchange rate was the topic of many research papers in the last years. However, evidence is quite mixed. For a discussion about the relationship between interest rates and exchange rates see Hnatkovska, Lahiri and Vegh (2008).

²¹We use the same interest rates as before, namely money market rates and government bond yields in a reduced sample from 1999:1 - 2007:1. Results are available in the unpublished web appendix.

²²The importance of tax regimes is reflected in the literature on structural breaks and regime-shifts in the analysis of the unit root properties of interest rates (as in Lai (2004)).

- Cheung, Y.-W., Tam, D. C. and Yiu, M. S.** (2008), "Does the Chinese interest rate follow the US interest rate?" *International Journal of Finance & Economics*, **13**(1), 53–67.
- Cheung, Y.-W. and Westermann, F.** (2002), "Output Dynamics of the G7 Countries–Stochastic Trends and Cyclical Movements". *Applied Economics*, **34**(18), 2239–47.
- Chinn, M. D. and Meredith, G.** (2004), "Monetary Policy and Long-Horizon Uncovered Interest Parity". *IMF Staff Papers*, **51**(3), 409–430.
- Choi, I. and Chul Ahn, B.** (1999), "Testing the null of stationarity for multiple time series". *Journal of Econometrics*, **88**(1), 41–77.
- Cubadda, G.** (1999), "Common Cycles in Seasonal Non-stationary Time Series". *Journal of Applied Econometrics*, **14**(3), 273–291.
- Engle, R. F. and Kozicki, S.** (1993), "Testing for Common Features". *Journal of Business & Economic Statistics*, **11**(4), 369–380.
- Gourieroux, C. and Peaucelle, I.** (1989), "Detecting a long run relationship (with an application to the p.p.p. hypothesis)". CEPREMAP Working Papers (Couverture Orange) 8902, CEPREMAP.
- Hnatkovska, V., Lahiri, A. and Vegh, C. A.** (2008), "Interest Rates and the Exchange Rate: A Non-Monotonic Tale". NBER Working Papers 13925, National Bureau of Economic Research, Inc.
- Johansen, S.** (1988), "Statistical analysis of cointegration vectors". *Journal of Economic Dynamics and Control*, **12**(2-3), 231–254.
- Johansen, S.** (1991), "Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models". *Econometrica*, **59**(6), 1551–1580.
- Karanasos, M., Sekioua, S. and Zeng, N.** (2006), "On the order of integration of monthly US ex-ante and ex-post real interest rates: New evidence from over a century of data". *Economics Letters*, **90**(2), 163–169.
- King, R. G., Plosser, C. I., Stock, J. H. et al.** (1991), "Stochastic Trends and Economic Fluctuations". *American Economic Review*, **81**(4), 819–840.
- Koustas, Z. and Lamarche, J.-F.** (2007), "Evidence of nonlinear mean reversion in the real interest rate". *Applied Economics*, **99999**(1), 1–12.
- Kugler, P. and Neusser, K.** (1993), "International Real Interest Rate Equalization: A Multivariate Time-Series Approach". *Journal of Applied Econometrics*, **8**(2), 163–174.
- Lai, K. S.** (2004), "On structural shifts and stationarity of the ex ante real interest rate". *International Review of Economics & Finance*, **13**(2), 217–228.
- MacKinnon, J. G.** (1996), "Numerical Distribution Functions for Unit Root and Cointegration Tests". *Journal of Applied Econometrics*, **11**(6), 601–618.

- Osterwald-Lenum, M.** (1992), "A Note with Quantiles of the Asymptotic Distribution of the Maximum Likelihood Cointegration Rank Test Statistics". *Oxford Bulletin of Economics and Statistics*, **54**(3), 461–472.
- Poghosyan, T. and de Haan, J.** (2007), "Interest Rate Linkages in EMU Countries: A Rolling Threshold Vector Error-Correction Approach". CESifo Working Paper Series 2060, CESifo GmbH.
- Rapach, D. E. and Weber, C. E.** (2004), "Are real interest rates really nonstationary? New evidence from tests with good size and power". *Journal of Macroeconomics*, **26**(3), 409–430.
- Romero-Ávila, D.** (2007), "Unit Roots and Persistence in the Nominal Interest Rate: A Confirmatory Analysis Applied to the OECD". *Canadian Journal of Economics*, **40**(3), 980–1007.
- Rose, A. K.** (1988), "Is the Real Interest Rate Stable?" *Journal of Finance*, **43**(5), 1095–1112.
- Schleicher, C.** (2007), "Codependence in cointegrated autoregressive models". *Journal of Applied Econometrics*, **22**(1), 137–159.
- Urga, G.** (2007), "Common Features in Economics and Finance: An Overview of Recent Developments". *Journal of Business & Economic Statistics*, **25**, 2–11.
- Vahid, F. and Engle, R. F.** (1997), "Codependent cycles". *Journal of Econometrics*, **80**(2), 199–221.
- Wu, Y. and Zhang, H.** (1996), "Mean Reversion in Interest Rates: New Evidence from a Panel of OECD Countries". *Journal of Money, Credit, and Banking*, **28**(4), 604–621.
- Zhou, S.** (2003), "Interest Rate Linkages within the European Monetary System: New Evidence Incorporating Long-Run Trends". *Journal of International Money and Finance*, **22**(4), 571–590.

Appendix

Table A.1: Data sources

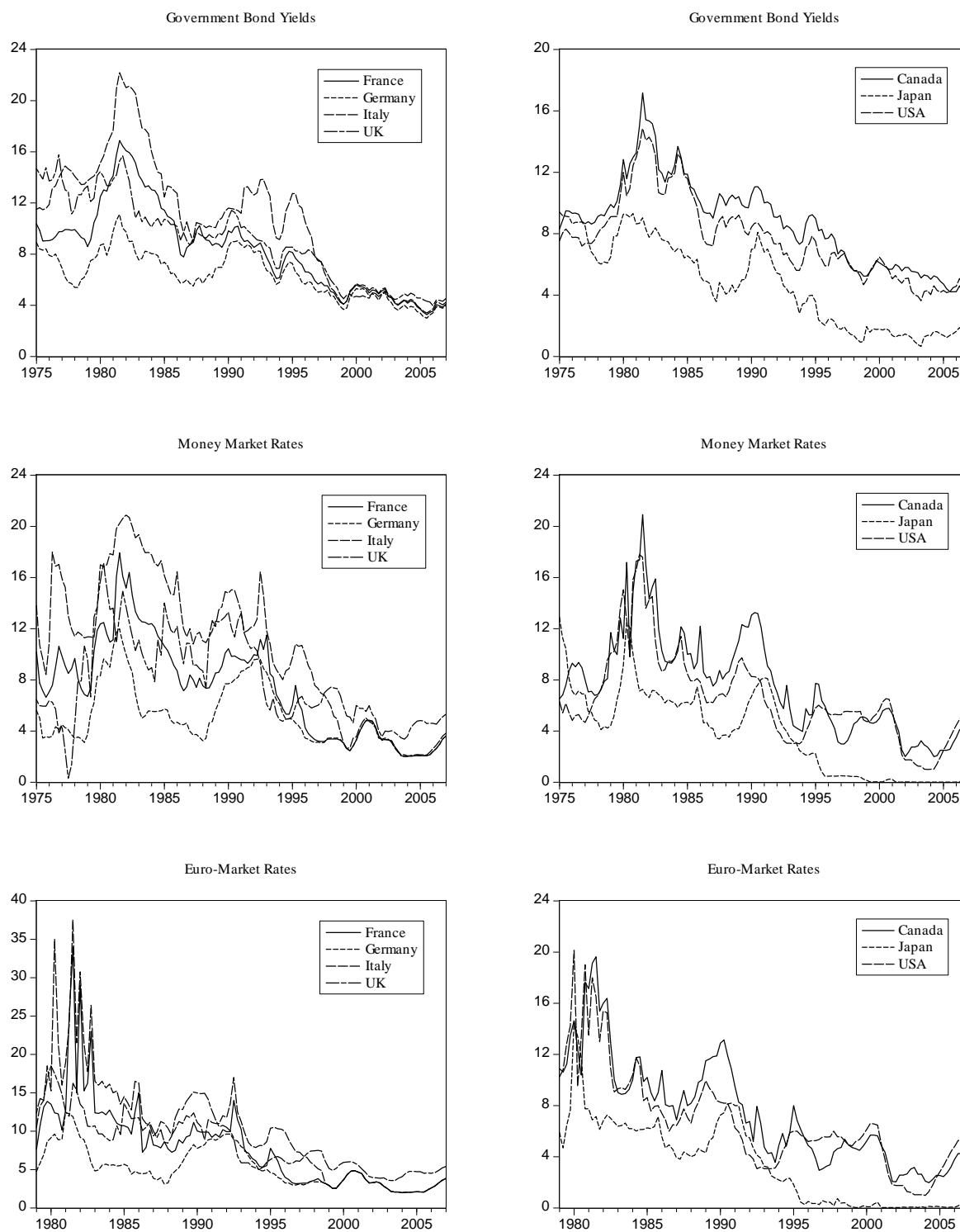
Country	Start	End	Source	Code
Money Market Rates				
Canada	1975q1	2007q1	IFS	15660B..ZF
France	1952q1	2007q1	Banque de France	mt.m.e00250.b.m.t.b.x
Germany	1957q1	2007q1	IFS	13460B..ZF
Italy	1971q1	2007q1	IFS	13660B..ZF
Japan	1957q1	2007q1	IFS	15860B..ZF
UK	1972q1	2007q1	IFS	11260B..ZF
USA	1957q1	2007q1	IFS	11160B..ZF
Government Bond Yields				
Canada	1957q1	2007q1	IFS	15661...ZF
France	1957q1	2007q1	IFS	13261...ZF
Germany	1957q1	2007q1	IFS	13461...ZF
Italy	1957q1	2007q1	IFS	13661...ZF
Japan	1966q4	2007q1	IFS	15861...ZF
UK	1957q1	2007q1	IFS	11261...ZF
USA	1957q1	2007q1	IFS	11161...ZF
Consumer Price Indexes				
Canada	1975q1	2007q1	IFS	15664...ZF
France	1957q1	2007q1	IFS	13264...ZF
Germany	1957q1	2007q1	IFS	13464.D.ZF + 13464...ZF
Italy	1957q1	2007q1	IFS	13664...ZF
Japan	1957q1	2007q1	IFS	15864...ZF
UK	1957q1	2007q1	IFS	11264...ZF
USA	1957q1	2007q1	IFS	11164...ZF
Euro-Market Rates				
Canada	1979m1	2008m2	Statistics Canada (STATCAN/CANSIMS)	D156RIBEUB1.D
France	1978m1	2008m2	Daily Press	D132RIBEUB1.D
Germany	1978m1	2008m2	Daily Press	D134RIBEUB1.D
Italy	1979m1	2008m2	Daily Press	D136RIBEUB1.D
Japan	1979m1	2008m2	Bank of Japan	D158RIBEUB1.D
UK	1978m1	2008m2	Financial Times	D112RIBEUB1.D
USA	1978m1	2008m2	Financial Times	D111RIBEUB1.D

Note: From 1992:1 the Real Interest Rates were calculated with the Inflation of reunified Germany. We thank Global Insight for kindly providing the Euro-Market rates.

Table A.2: Serial correlation common feature with all pairs of stationary real interest rates

1975:1 - 2007:1								
Common Feature Test of Real Money Market Rates								
		Canada	France	Germany	Italy	Japan	UK	USA
Italy	lags	8	--	--	--			
	Coef.	0.5057	--	--	--			
	F-Stat.	6.944 ***	--	--	--			
USA	lags	8	--	--	7	--	--	--
	Coef.	0.7582	--	--	0.9517	--	--	--
	F-Stat.	9.445 ***	--	--	46.15 ***	--	--	--
Common Feature Test of Real Government Bond Yields								
		Canada	France	Germany	Italy	Japan	UK	USA
France	lags	5	--					
	Coef.	0.9004	--					
	F-Stat.	23.66 ***	--					
Germany	lags	2	5	--				
	Coef.	1.1466	1.2009	--				
	F-Stat.	133.2 ***	84.81 ***	--				
Italy	lags	5	5	5	--			
	Coef.	0.5877	0.6351	0.1594	--			
	F-Stat.	30.91 ***	30.09 ***	43.17 ***	--			
UK	lags	8	8	8	8	--	--	
	Coef.	0.5535	0.5558	0.1685	0.8368	--	--	
	F-Stat.	27.84 ***	32.63 ***	31.65 ***	22.62 ***	--	--	
USA	lags	2	5	2	5	--	8	--
	Coef.	0.7616	0.7318	0.3118	0.9268	--	0.9512	--
	F-Stat.	129.3 ***	70.13 ***	96.85 ***	76.43 ***	--	30.54 ***	--

Note: Table A.2 reports common feature tests for each pair of stationary real interest rates for the sample 1975:1 to 2007:1. For each pair, the longer lag length of the two series is chosen for the common feature test (see table 1). The first row for each pair contains the lag length, the second row reports the coefficient of the common feature vector and the third row reports the F-statistic of the common feature test. *, ** and *** indicate rejection of the null hypothesis with a significance of 10%, 5% and 1%.

Figure 1: Nominal interest rates

Note: Nominal Interest Rates are displayed.